# Mathematics for Intelligent Systems Lecture 1 Homework
## (Linear Algebra I: Vector Spaces, Bases, Matrix Representations of Linear Transforms)

Marc Toussaint, Andrea Baisero

**Abstract**

This homework explores coordinate-free definitions of vector spaces, and cordinate-ful representations which arise only once a basis is chosen.

## 1 Problem 1

a. Find 3 interesting examples of vector spaces that we haven't covered in class. What are their dimensions? What bases are commonly used to represent them? (It's okay to look this up in a textbook or online—just find interesting examples.)

b. Come up with one example of a set which is *almost* a vector space, i.e. it satisfies some of the core requirements of a vector space, but not all.

c. Consider the set of of all *even* functions. Is this a Vector space? If so, define 3 different bases which span the entire space.

## 2 Problem 2

Consider a collection of functions $\mathcal{V}$ defined by

$$f(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \ldots + \alpha_n x^n \tag{1}$$

where $x \in \mathbb{R}$ and $\alpha_i \in \mathbb{R}$ for each $i = 0, \ldots, n$.

a. Is this collection of functions a vector space? Why?

b. Consider the bases

$$\mathcal{B}_1 = \{1,\ x,\ x^2,\ \ldots,\ x^n\}$$

and

$$\mathcal{B}_2 = \{1,\ 1+x,\ 1+x+x^2,\ \ldots,\ 1+x+\ldots+x^n\}.$$

Define a linear transform $\mathbf{T}$ that takes

$$\begin{aligned}
&1 \to 1 \\
&x \to 1+x \\
&x^2 \to 1+x+x^2 \\
&\vdots
\end{aligned}$$

This transform takes basis elements of $\mathcal{B}_1$ directly to basis elements of $\mathcal{B}_2$. What is the matrix for the linear transformation in these bases?

c. Let $f(x) = 1+x+x^2+x^3$. (This function $f$ is a vector in the vector space $\mathcal{V}$, so from here on out we refer to it as a vector rather than a function.) What are the coefficients of this vector in terms of $\mathcal{B}_1$? What are the coefficients of the transformed vector in terms of $\mathcal{B}_2$? Are these latter coefficients the ones you get from multiplying the former coefficients by the matrix you derived above?

d. What would the matrix be if the range basis were $\mathcal{B}_1$ rather than $\mathcal{B}_2$? (Remember that the “range” space is the output space of the mapping.)