Machine Learning
Exercise 5

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June 15, 2013

1 Independence

Write an algorithm (Octave or just pseudo code) that tests, for any given joint probability table $P(X, Y)$ over 2 random variables, whether the two random variables are independent. Test your algorithm on the following table:

	Y=0	Y=1	Y=2
X=0	.08	.12	.2
X=1	.12	.18	.3

(When writing pseudo code, be explicit what type/size all objects are.)

2 Sum of 3 dices

You have 3 dices (potentially fake dices where each one has a different probability table over the 6 values). You're given all three probability tables $P(D_1)$, $P(D_2)$, and $P(D_3)$. Write an algorithm that computes the conditional probability $P(S|D_1)$ of the sum of all three dices conditioned on the value of the first dice.

3 Conditionalized versions of product and Bayes rule

Prove from first principles the conditionalized version of the product rule (the same as the product rule, but every term is additionally conditioned on Z):

$$P(X, Y|Z) = P(Y|X, Z)\ P(X|Z)\ .$$

Also prove the conditionalized version of Bayes rule

$$P(X|Y, Z) = \frac{P(Y|X, Z)P(X|Z)}{P(Y|Z)}\ .$$

4 Product of Gaussians

Slide 06:25 defines a 1D and n-dimensional Gaussian distribution. See also Wikipedia, if necessary. Multiplying probability distributions is a fundamental operation, and multiplying two Gaussians is needed in many models. From the definition of a n-dimensional Gaussian, prove the general rule

$$\mathcal{N}(x\,|\,a, A)\ \mathcal{N}(x\,|\,b, B)\ \propto\ \mathcal{N}(x\,|\,(A^{-1} + B^{-1})^{-1}(A^{-1}a + B^{-1}b), (A^{-1} + B^{-1})^{-1})\ ,$$

where the proportionality $\propto$ allows you to drop all terms independent of x.

Note: The so-called canonical form of a Gaussian is defined as $\mathcal{N}[x\,|\,\bar a, \bar A] = \mathcal{N}(x\,|\,\bar A^{-1}\bar a, \bar A^{1})$; in this convention the product reads much nicher: $\mathcal{N}[x\,|\,\bar a, \bar A]\ \mathcal{N}[x\,|\,\bar b, \bar B] \propto \mathcal{N}[x\,|\,\bar a + \bar b, \bar A + \bar B]$. You can first prove this before proving the above, if you like.