University of Stuttgart

Reinforcement Learning

Temporal difference learning

Daniel Hennes

29.05.2017

University Stuttgart - IPVS - Machine Learning & Robotics

Temporal difference (TD) learning

- Incremental Monte Carlo
- TD prediction
- TD vs MC vs DP
- TD for control:
  - SARSA
  - Q-learning
  - Expected SARSA

Incremental Monte Carlo algorithm

- First–visit MC:
  - G_t is the return following our first visit to $s = S_t$ in time t
  - append G_t to $Returns(s)$
  - $V(s) = average(Returns(s))$
- We can implement this as an incremental update:

$$V(s) \leftarrow V(s) + \frac{1}{n(s)}\big[G_t - V(s)\big]$$

where $n(s)$ is the number of first visits to s

Incremental Monte Carlo algorithm

- We can also formulate a constant–α Monte Carlo update:

$$V(S_t) \leftarrow V(S_t) + \alpha\big[G_t - V(S_t)\big]$$

- Why is this useful when tracking a non-stationary problem?

Temporal difference prediction

- Policy evaluation is often referred to as the *prediction problem*
- We have some policy π which tells the agent which action a to choose in state s
- Find the value function $v_\pi(s)$ of this policy, i.e. **evaluate** the policy π

$$V(S_t) = V(S_t) + \alpha\big[\underbrace{G_t}_{\text{MC target}} - V(S_t)\big]$$

- Simplest temporal difference update TD(0):

$$V(S_t) = V(S_t) + \alpha\Big[\underbrace{\underbrace{R_{t+1} + \gamma V(S_{t+1})}_{\text{TD target}} - V(S_t)}_{\text{TD error}}\Big]$$

- Reinforcement:
  - more reward than expected: $R_{t+1} + \gamma V(S_{t+1}) > V(S_t) \Rightarrow V(S_t) \uparrow$
  - less reward than expected: $R_{t+1} + \gamma V(S_{t+1}) < V(S_t) \Rightarrow V(S_t) \downarrow$

Temporal difference learning

TD combines **sampling** of Monte Carlo with **bootstrapping** from dynamic programming:

$$
\begin{aligned}
v_\pi(s) = & \underbrace{\mathbb{E}_\pi\left[G_t \mid S_t = s\right]}_{\text{MC target uses sample return}} \\
& = \mathbb{E}_\pi\left[R_{t+1} + \gamma G_{t+1} \mid S_t = s\right] \\
& = \underbrace{\mathbb{E}_\pi\left[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s\right]}_{\text{DP target uses current estimate } V(S_{t+1})}
\end{aligned}
$$

- TD(0) update:

$$V(S_t) \leftarrow V(S_t) + \alpha\big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\big]$$

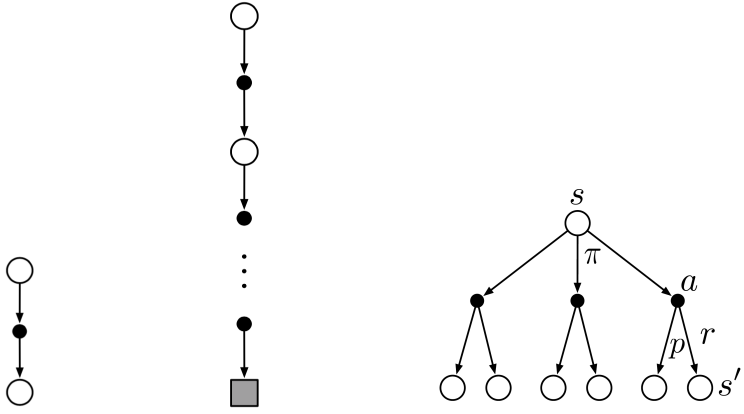
- Dynamic programming update:

$$V(s) \leftarrow \sum_a \pi(s,a) \sum_{s',r} p(s', r \mid s, a)\Big[r + \gamma V(s')\Big] \text{ for all } s \in \mathcal{S}$$

Bootstrapping and sampling

- **Bootstrapping**
  - TD updates its estimates of $v_\pi(s)$ based on other estimates of v_π
  - DP also uses bootstrapping
  - MC does not use bootstrapping; estimates returns at end of episode
- **Sampling**
  - TD updates are based on sampled paths through the state space
  - MC also samples
  - DP does not sample; updates estimates based on all states that can be reached

Backup diagrams

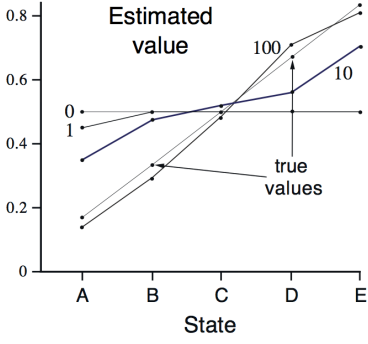


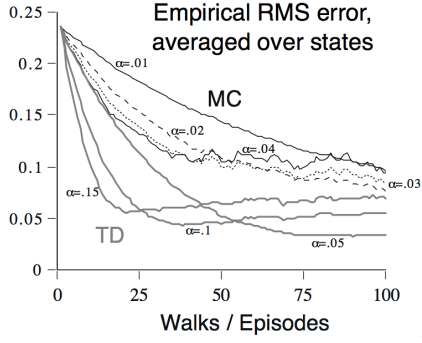
TD prediction versus MC prediction

- TD can learn before episode termination
- TD can be used for either non–episodic or episodic tasks
- Update depends on single stochastic transition: *lower variance*
- Updates use bootstrapping: estimate has a *bias*
- TD updates exploit the Markov property

- MC learning must wait until the episode terminate
- MC only works for episodic tasks
- Update depends on a sequence of many stochastic transitions: *larger variance*
- MC is an *unbiased* estimate
- MC updates does not exploit the Markov property; can be effective in non–Markovian environments

Random walk example





The value of each state is a prediction of the probability of terminating on the right

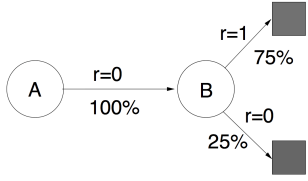
$$V(\mathrm{A}) = \tfrac{1}{6}, V(\mathrm{B}) = \tfrac{2}{6}, V(\mathrm{C}) = \tfrac{1}{2} \ldots$$

Difference between TD and MC estimates

- Suppose we observe the following 8 episodes from a Markov reward process:
  1. A, 0, B, 0
  2. B, 1
  3. B, 1
  4. B, 1
  5. B, 1
  6. B, 1
  7. B, 1
  8. B, 0
- First episode starts in state A, transitions to B getting reward of 0, and terminates with a reward of 0
- Second episode starts in state B and terminates with reward of 1
- ...
- What are the best estimates for the values $V(\mathrm{A})$ and $V(\mathrm{B})$?

Modelling the underlying Markov process

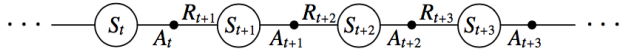
- Option 1: Monte Carlo
  - B terminates with 1 in 6/8 cases and with 0 in 2/8 cases $\Rightarrow V(\text{B}) = 0.75$
  - Only episode starting from A terminates with 0 $\Rightarrow V(\text{A}) = 0$
- Option 2: modelling the underlying Markov process
  - A transitions to B in 100% of the cases
  - B has value of 0.75, with $\gamma = 1 \Rightarrow V(\text{A}) = 0.75$



TD and MC batch estimates

- Batch Monte Carlo (updating after all episodes are done) results in $V(A) = 0$
  - best match for the training data
  - minimises the mean-square error on the training set
- Consider sequentiality, i.e. A goes to B goes to terminating state; then $V(A) = 0.75$
  - this is what batch $TD(0)$ does
  - correct maximum–likelihood estimate of the model (Markov reward process)
  - assumes model is correct and estimates the value function: *certainty-equivalence estimate*
  - we expect that this will produce better estimates for future data

TD for control



- Estimate action–value function instead of state–value function
- Estimate $q_\pi(s, a)$ for the current behavior policy π (on–policy)

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\big[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\big]$$

- Update after every transition from a *non–terminal* state S_t
  - $Q(S_{t+1}, \cdot) = 0$ if S_{t+1} is *terminal*

On–policy TD control with Sarsa

- Choose a behaviour policy π and estimate the Q-values (Q_π) using the Sarsa update rule

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\big[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\big]$$

- Change π towards greediness wrt Q_π
- Use soft policies, e.g. ϵ–greedy
- Converges with probability 1 to optimal policy and action–values if it visits all state–action pairs infinitely many times and policy converges to greedy policy, e.g. by arranging for $\epsilon \to 0$
  - Greedy in the limit with infinite exploration (GLIE)
  - Stochastic approximation conditions: $\sum_{t=1}^{\infty} \alpha_t(s,a) = \infty$ and $\sum_{t=1}^{\infty} \alpha_t^2(s,a) < \infty$
- Learning optimal action–values is of course useful for control since it tells us immediately which is (or are) the optimal action(s)

Sarsa (on–policy TD control)

Initialize $Q(s, a)$, for all $s \in \mathcal{S}, a \in \mathcal{A}$, arbitrarily, and $Q(terminal - state, \cdot) = 0$

for each episode **do**

  Initialize S

  Choose A from S using policy derived from Q (e.g., ϵ–greedy)

  repeat(for each step of episode)

    Take action A, observe R, S'

    Choose A' from S' using policy derived from Q (e.g., ϵ–greedy)

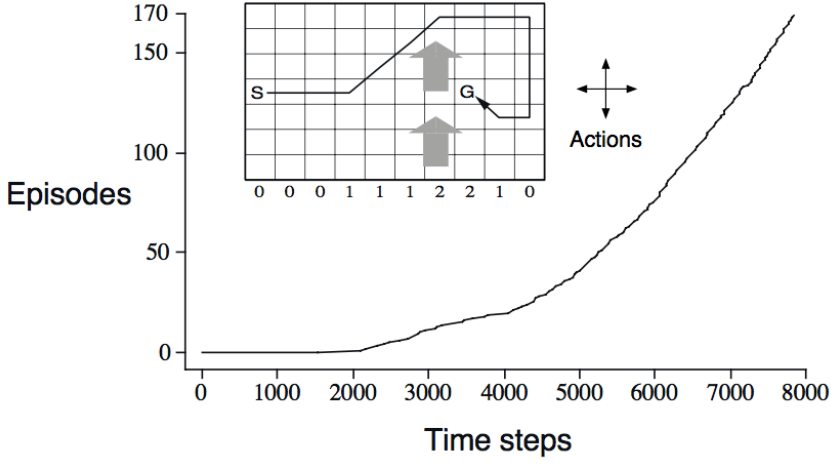
    $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma Q(S', A') - Q(S, A)]$

    $S \leftarrow S'; A \leftarrow A'$

  until S is terminal

end for

Windy gridworld example



Axis of ordinates: episodes = cumulative number of times the goal has been reached

Q–learning

- Sarsa is an example of *on–policy* learning
- Q–learning is an *off–policy* TD control algorithm

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\big]$$

- Always update using maximum Q–value available from next state
- **Target policy:** is the greedy policy: $\pi(a) = \arg\max_a Q(s, a)$
- **Behavior policy:** soft policy, e.g. ϵ–greedy, $\epsilon > 0$
- *Off–policy* learning: target policy $\neq$ behavior policy

Q–learning (off–policy TD control)

Initialize $Q(s, a)$, for all $s \in \mathcal{S}, a \in \mathcal{A}$, arbitrarily, and $Q(\textit{terminal} - \textit{state}, \cdot) = 0$

for each episode **do**

  Initialize S

  repeat(for each step of episode)

    Choose A from S using policy derived from Q (e.g., ϵ–greedy)

    Take action A, observe R, S'

    $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$

    $S \leftarrow S'$

  until S is terminal

end for

Backup diagrams for Sarsa and Q-learning

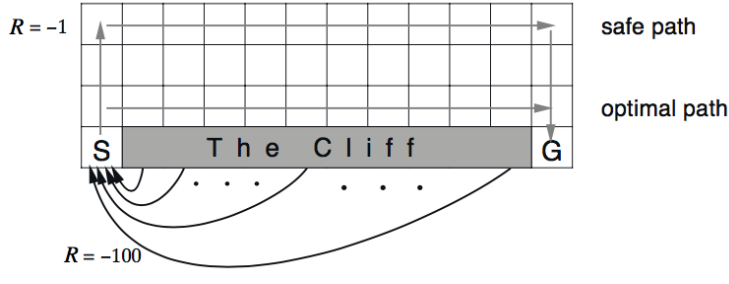
- Sarsa backs up using the actual action chosen by the behavior policy

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\big[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\big]$$

- Q–learning backs up using the best next action (i.e. highest value under current estimates)

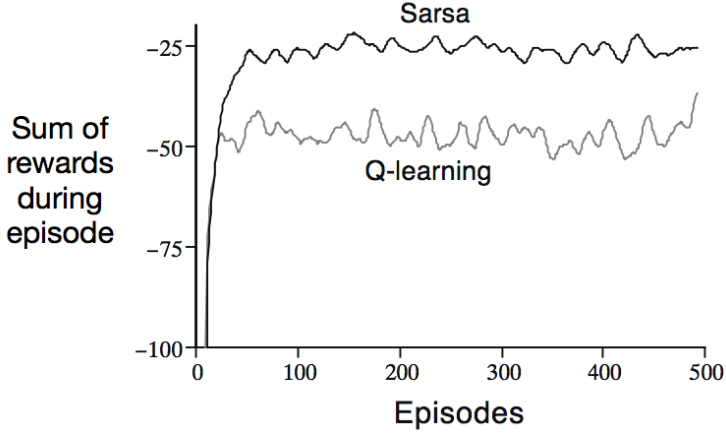
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\big]$$

Cliff walk example



- Deterministic transitions (actions: up, down, left, right)
- ϵ–greedy action selection ($\epsilon = 0.1$)

Cliff walk example



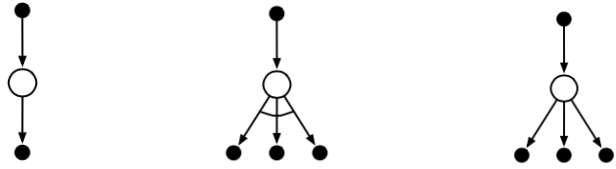
Expected Sarsa

- Just like Q–learning, but *expectation* instead of *maximization*

$$\begin{aligned} Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \alpha\big[R_{t+1} + \gamma\, \mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t)\big] \\ &\leftarrow Q(S_t, A_t) + \alpha\big[R_{t+1} + \gamma \sum_a \pi(a \mid S_{t+1}) Q(S_{t+1}, a) - Q(S_t, A_t)\big] \end{aligned}$$

- *Deterministically* moves in the same direction as Sarsa moves *in expectation*
- Given same amount of experience it generally performs slightly better than Sarsa
- Can be used off–policy, where the behavior policy differs from π
- What if the behavior policy is exploring and π is greedy wrt to current estimates?

Backup diagrams for Sarsa, Q-learning, Expected Sarsa



Maximization bias

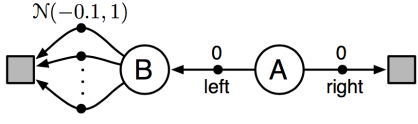
- Many control algorithms involve maximization in the construction of their target policy

$$\pi(s) = \arg\max_a Q(s, a)$$

- Maximum over estimated action–values is used implicitly as an estimate of the maximum value:

$$\max_a \mathbb{E}[Q(s, a)] \neq \mathbb{E}[max_a Q(s, a)]$$

- This can lead to a significant *positive maximization bias*

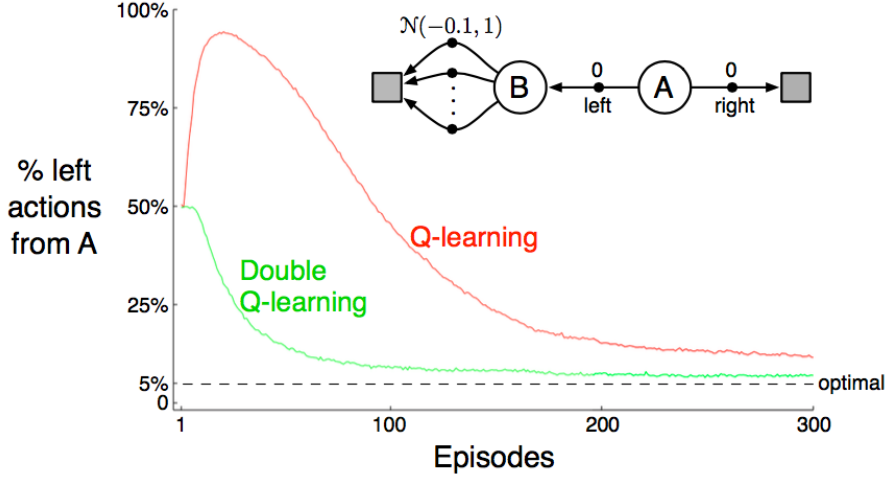


Double Q-learning

- Learn two independent estimates $Q_1(s, a)$ and $Q_2(s, a)$
- Divide time steps in two (flipping a coin), update only one estimate at each time step

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha\big[R_{t+1} + \gamma Q_2(S_{t+1}, \arg\max_a Q_1(S_{t+1}, a)) - Q_1(S_t, A_t)\big]$$

- The two approximate value functions are treated completely symmetrical (i.e. switch Q_1 for Q_2 for other side of coin)
- Behavior policy may use both approximations, e.g. ϵ–greedy wrt $Q_1 + Q_2$


100%
75%
50%
25%
5%
0
% left actions from A
$\mathcal{N}(-0.1, 1)$
B
A
0
left
0
right
Q-learning
Double Q-learning
optimal
1
100
200
300
Episodes

Summary

- TD is an alternative to MC for solving the prediction problem
- Extension to control problem via GPI (prediction + local policy improvement)
- Here prediction is based on actual *experience*
  - we need to maintain sufficient exploration
  - *on–* vs. *off–policy* learning
- Sarsa (on–policy), Q–learning (off–policy), Expected Sarsa (both)