Robotics
Exercise 10

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1 Kalman Localization (8 Points)

We consider the same car example as for the last exercise, but track the car using a Kalman filter.
Access the code as usual:

1. To make sure you have an updated version of the repository, run 'git pull' and 'git submodule update'

2. For python run: 'jupyter-notebook py/08-kalman/08-kalman.ipynb'
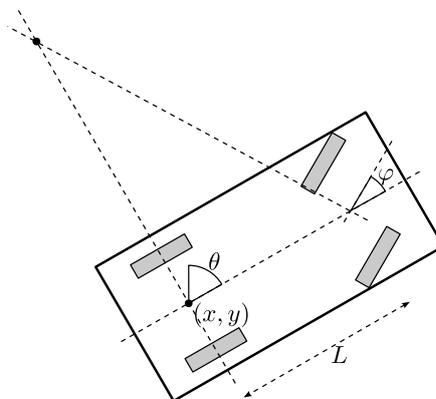
3. For C++ run: 'cd cpp/08-kalman', 'make', './x.exe'

The motion of the car is described by the following:

State $q = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$ **Controls** $u = \begin{pmatrix} v \\ \varphi \end{pmatrix}$

Sytem equation $\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = f(q, u) = \begin{pmatrix} v \; \cos\theta \\ v \; \sin\theta \\ (v/L) \; \tan\varphi \end{pmatrix}$

$|\varphi| < \Phi$

(In the following, we will denote the state with x_t (instead of q_t), such that it is consistent with the lecture slides.)

a) To apply a Kalman filter (slide 06:29) we need Gaussian models for $P(x_t \,|\, x_{t\text{-}1}, u_{t\text{-}1})$ as well as $P(y_t|x_t)$. We assume that the dynamics model is given as a local Gaussian of the form

$$P(x_{t+1} \,|\, x_t, u_t) = \mathcal{N}(x_{t+1}|x_t + B(x_t)u_t), \sigma_{\text{dynamics}})$$

where the matrix $B(x_t) = \frac{\partial f(x_t, u_t)}{\partial u_t}$ gives the local linearization of the car dynamics. What is $B(x_t)$ (the Jacobian of the state change w.r.t. u) for the car dynamics? (2P)

b) Implement the linearized dynamics model in the function 'getControlJacobian()'. (2P)

c) Concerning the observation likelihood $P(y_t|x_t)$ we assume

$$P(y_t|x_t, \theta_{1:N}) = \mathcal{N}(y_t \,|\, C(x_t)x_t + c(x_t), \sigma_{\text{observation}})$$

What is the matrix $C(x_t)$ (the Jacobian of the landmark positions w.r.t. the car state) in our example? (2P)

Hints:

- Assume there is only one landmark in the world.
- The car observes this landmark in its own coordinate frame, $y = l^C \in \mathbb{R}^2$.
- Write down the transformations between world and car coordinates $T_{W \to C}$ and the inverse $T_{C \to W}$.
- Use them to define a mapping $y = g(x_t)$ that maps the state x_t to the observation y.
- Compute the local linearization $C = \frac{\partial g(x_t)}{\partial x_t}$.

d) Implement the Kalman filter (slide 06:29) to track the car (this does not require a solution to question part c).
Note that $c(x_t) = \widehat{y}_t - C(x_t)x_t$, where $\widehat{y}_t$ is the mean observation in state x_t. The variables $C, A, Q, W, \widehat{y}_t$ of the Kalman filter are already provided in the code. (2P)

Tips:

– ˜X transposes the matrix X

– `inverse(X)` calculates the matrix inverse of matrix X

2   Bayes Basics (4 Points)

a) Box 1 contains 8 apples and 4 oranges. Box 2 contains 10 apples and 2 oranges. Boxes are chosen with equal probability. What is the probability of choosing an apple? If an apple is chosen, what is the probability that it came from box 1? (1P)

b) The blue M&M was introduced in 1995. Before then, the color mix in a bag of plain M&Ms was: 30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan. Afterward it was: 24% Blue , 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown.
A friend of mine has two bags of M&Ms, and he tells me that one is from 1994 and one from 1996. He won't tell me which is which, but he gives me one M&M from each bag. One is yellow and one is green. What is the probability that the yellow M&M came from the 1994 bag? (1P)

c) The Monty Hall Problem: I have three boxes. In one I put a prize, and two are empty. I then mix up the boxes. You want to pick the box with the prize in it. You choose one box. I then randomly select *another* one of the two remaining boxes and show that it is empty (this is not supposed to be another random experiment; just assume that an empty box is picked). I then give you the chance to change your choice of boxes—should you do so? (1P)

d) Given a joint probability $P(X, Y)$ over 2 binary random variables as the table

	Y=0	Y=1
X=0	.06	.24
X=1	.14	.56

What are $P(X)$ and $P(Y)$? Are X and Y independent? (1P)