

Robotics

Exercise 1

Marc Toussaint

Lecturer: Duy Nguyen-Tuong

TAs: Philipp Kratzer, Janik Hager, Yoojin Oh

Machine Learning & Robotics lab, U Stuttgart

Universitätsstraße 38, 70569 Stuttgart, Germany

October 16, 2019

1 Matrix equations (4 points)

a) Let X, A be arbitrary matrices, A invertible. Solve for X :

$$XA + A^T = \mathbf{I}$$

b) Let X, A, B be arbitrary matrices, $(C - 2A^T)$ invertible. Solve for X :

$$X^T C = [2A(X + B)]^T$$

c) Let $x \in \mathbb{R}^n, y \in \mathbb{R}^d, A \in \mathbb{R}^{d \times n}$. A obviously *not* invertible, but let $A^T A$ be invertible. Solve for x :

$$(Ax - y)^T A = \mathbf{0}_n^T$$

d) As above, additionally $B \in \mathbb{R}^{n \times n}$, B positive-definite. Solve for x :

$$(Ax - y)^T A + x^T B = \mathbf{0}_n^T$$

2 Vector derivatives (5 points)

Let $x \in \mathbb{R}^n, y \in \mathbb{R}^d, f, g : \mathbb{R}^n \rightarrow \mathbb{R}^d, A \in \mathbb{R}^{d \times n}, C \in \mathbb{R}^{d \times d}$. (Also provide the dimensionality of the results.)

a) What is $\frac{\partial}{\partial x} x$?

b) What is $\frac{\partial}{\partial x} [x^T x]$?

c) What is $\frac{\partial}{\partial x} [f(x)^T f(x)]$?

d) What is $\frac{\partial}{\partial x} [f(x)^T C g(x)]$?

e) Let B and C be symmetric (and pos.def.). What is the minimum of $(Ax - y)^T C (Ax - y) + x^T B x$?

3 Optimization (3 points)

Given $x \in \mathbb{R}^n, f : \mathbb{R}^n \rightarrow \mathbb{R}$, we want to find $\operatorname{argmin}_x f(x)$. (We assume f is uni-modal.)

a) What 1st-order optimization methods (querying $f(x), \nabla f(x)$ in each iteration) do you know?

b) What 2nd-order optimization methods (querying $f(x), \nabla f(x), \nabla^2 f(x)$ in each iteration) do you know?

c) What is backtracking line search?