

Robotics

Exercise 6

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1 Riccati equation in discrete time (5 points)

Consider the time discrete linear quadratic system

$$f(x_t, u_t) = Ax_t + Bu_t$$

$$c(x_t, u_t) = x_t^\top Qx_t + u_t^\top Ru_t$$

with the cost function

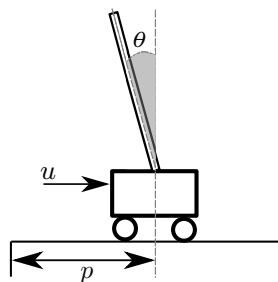
$$J^\pi = \sum_{t=0}^{\infty} c(x_t, u_t) .$$

The Bellman equation (slide 04:15) for this infinite-horizon discrete time system is

$$V(x) = \min_u [c(x, u) + V(f(x, u))] .$$

Start with the Bellman equation and derive the Riccati equation for the system. Similar to the continuous case, you can assume a value function of the form $V(x) = x^\top Px$ with a symmetric matrix P .

2 Cart Pole Control (7 points)



In the last exercise we calculated the local linearization of the cart-pole around $x^* = (0, 0, 0, 0)$. The solution is

$$\dot{x} = Ax + Bu, \quad A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{c_2 g}{\frac{4}{3}l - c_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{\frac{4}{3}l - c_2} & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ c_1 + \frac{c_1 c_2}{\frac{4}{3}l - c_2} \\ 0 \\ \frac{-c_1}{\frac{4}{3}l - c_2} \end{pmatrix}$$

with $g = 9.8ms^2$ the gravitational constant, $l = 1m$ the pendulum length and constants $c_1 = (M_p + M_c)^{-1}$ and $c_2 = lM_p(M_p + M_c)^{-1}$ where $M_p = M_c = 1kg$ are the pendulum and cart masses respectively.

a) We assume a stationary infinite-horizon cost function of the form

$$J^\pi = \int_0^\infty c(x(t), u(t)) dt$$

$$c(x, u) = x^\top Qx + u^\top Ru$$

$$Q = \text{diag}(c, 0, 1, 0), \quad R = \mathbf{I}.$$

That is, we penalize position offset $c\|p\|^2$ and pole angle offset $\varrho\|\theta\|^2$. Choose $c = \varrho = 1$ to start with.

Solve the Algebraic Riccati equation

$$0 = A^\top P + P^\top A - PBR^{-1}B^\top P + Q$$

by initializing $P = Q$ and iterating using the following iteration:

$$P_{k+1} = P_k + \epsilon[A^\top P_k + P_k^\top A - P_k B R^{-1} B^\top P_k + Q]$$

Choose $\epsilon = 1/1000$ and iterate until convergence (at least 10.000 iterations). Output the gains $K = -R^{-1}B^\top P$. (3 P)

b) Solve the same Algebraic Riccati equation analytically using some math library (python, octave, matlab, ...).

For Python, install `scipy` (using `'python3 -m pip install scipy --user'`), use `from scipy import linalg` to import `scipy` in the python script and use `P=linalg.solve_continuous_are(A,B,Q,R)` to solve the ARE. (2 P)

(The solution is $K = (1.0000, 2.6088, 52.9484, 16.5952)$.)

c) Implement the optimal Linear Quadratic Regulator $u^* = Kx$ on the cart pole simulator in the function `testMove()`.

1. For python please install `pygame` and `pyopengl` (using `'python3 -m pip install pygame pyopengl --user'`), then you can run: `'jupyter-notebook py/04-riccati/04-riccati.ipynb'`
2. For C++ run: `'cd cpp/04-riccati', 'make', './x.exe'`

Simulate the optimal LQR and test it for various noise levels (by changing the variable `dynamicsNoise`). (2 P)