Robotics
Exercise 7

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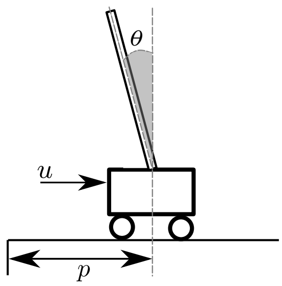
1 Stability (12 points)



In the last exercise we calculated the local linearization of the cart-pole around $x^* = (0, 0, 0, 0)$. The solution is

$$\dot{x} = Ax + Bu \ , \quad A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{c_2 g}{\frac{4}{3}l - c_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{\frac{4}{3}l - c_2} & 0 \end{pmatrix} , \quad B = \begin{pmatrix} 0 \\ c_1 + \frac{c_1 c_2}{\frac{4}{3}l - c_2} \\ 0 \\ \frac{-c_1}{\frac{4}{3}l - c_2} \end{pmatrix}$$

with $g = 9.8ms^2$ the gravitational constant, $l = 1m$ the pendulum length and constants $c_1 = (M_p + M_c)^{-1}$ and $c_2 = lM_p(M_p + M_c)^{-1}$ where $M_p = M_c = 1kg$ are the pendulum and cart masses respectively.

a) Consider the local linearization of the cart-pole. Is the system controllable? (2 P)

b) Consider the *uncontrolled* system (where there are no controls, $u = 0$). Perform a linear stability analysis. (Show whether the system is asymptotically stable, marginally stable, or unstable.) (2 P)

c) Consider a linear controller $u = w^\top x$ with 4 parameters $w \in \mathbb{R}^4$ for the cart-pole. What is the closed-loop linear dynamics $\dot{x} = \widehat{A}x$ of the system? (2 P)

d) Test if the controller with $w = (1.0000, 2.6088, 52.9484, 16.5952)$ (computed using ARE) is asymtotically stable. What are the eigenvalues? (2 P)

e) Given some eigenvalues $\lambda_1^\star, \lambda_2^\star, \lambda_3^\star, \lambda_4^\star$. Come up with a method that finds parameters w for these eigenvalues around $x^* = (0, 0, 0, 0)$. What could "good" eigenvalues be to achieve a "maximally stable" system (e.g., asymptotically stable with fastest convergence rate)? (2 P)

f) Output the optimal parameters and test them on the cart-pole simulation. (2 P)

1. For python please install pygame and pyopengl (using *' python3 -m pip install pygame'* and *' python3 -m pip install pyopengl'*), then you can run: *'jupyter-notebook py/05-stability/05-stability.ipynb'*

2. For C++ run: *'cd cpp/05-stability'*, *'make'*, *'./x.exe'*